On themomagnetic effects due to the supeconducting fluctuations: Reply to arXiv:1012.4361 by Serbyn, Skvortsov, and Varlamov

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In a recent Reply [1] to our Comment [2], Serbyn, Skvortsov, and Varlamov raised a question of microscopic description, which we did not touch in [2], and criticized our work [3]. They hopefully agreed with one of our key result [3] that the effective heat current vertex for fluctuating Cooper pairs (the Aslamzov-Larkin block in the diagram technique) is modified in the magnetic field, so "the heat current is proportional to the gauge-invariant momentum" [1] . However, they stated that in [3] we have overlooked the same correction to the electric current vertex and in this way we lost their huge thermomagnetic effect that does not require any particle-hole asymmetry and, therefore, prevails over the ordinary thermomagnetic effects by at least *five orders in magnitude* in ordinary superconductors near T_c and strongly dominates in the temperature range up to $\sim 100T_c$. Here we address their criticism with all details and show that our calculations in [3] are correct.

PACS numbers:

In our Comment [2] on "Giant Nernst Effect due to Fluctuating Cooper Pairs in Superconductors" [4], we highlighted that the magnetization currents do not transfer the heat. Therefore, the amendment of the heat current by "*circular magnetization heat current*, $\mathbf{j}^Q_M = c(\mathbf{M}\times\mathbf{E})$," ($\mathbf{M}$ is the magnetization) that was used in [4] is wrong. In fact, the term $(\mathbf{M}\times\mathbf{E})$ is the *magnetization part of the Poynting vector* [2]. Without such correction the results of [4] contradict the third law of thermodynamics. Besides this, we also stressed [2] that the Gaussian model is fully applicable to ordinary superconductors, for which [4] predicts the fluctuation correction to β to be at least $\epsilon_F/T \sim 10^5$ times bigger than β for noninteracting electrons. Moreover, far above T_c this huge effect was predicted to decrease as T^{-2} and, therefore, it would dominate in the wide temperature range up to $\sim 100T_c$, i.e. up to the room temperatures [5]. Certainly, such huge effects are not known for ordinary superconductors (Nb, Al, Sn, and etc), which just slightly above T_c show the same values and temperature dependencies as nonsuperconducting metals. It is also well understood that large thermomagnetic effects are observed in materials with small Fermi energy, i.e. with the large particle-hole asymmetry [6,7].

In their Reply [1], Serbyn, Skvortsov, and Varlamov did not address our general objections [2] to their Letter [4] and instead criticized our microscopic calculations in the previous paper [3].

The key result of our work [3] is that the heat current vertex for fluctuating Cooper pairs (the Aslamzov-Larkin block in the diagram technique, see Fig. 1) is modified by the magnetic field,

$$\mathbf{B}^h = \frac{\omega}{2e}B\left(\mathbf{q}+\frac{2e}{c}\mathbf{A}^H\right), \tag{1}$$

where (ω, q) are the energy and momentum of Cooper pairs, near the transition B is some constant, and $\mathbf{A}^H$ is the vector potential of the magnetic field.

Our opponent agree that we *"correctly obtain that the heat current is proportional to the gauge-invariant momentum"* [1] (see Eq. 2 in [1]).

At the same time, our opponents stated that we *"fail to include $\mathbf{A}^H$ in the electric vertex and draw the diagrams, extracting $\mathbf{A}^H$ from the propagators and from the heat vertex only"* [1]. Obviously, they assumed the electric current vertex in the magnetic field has a form

$$\mathbf{B}^e = B\left(\mathbf{q}+\frac{2e}{c}\mathbf{A}^H\right). \tag{2}$$

They claimed that we did not take into account the second term in this equation.

Here we clarify our calculations. Below we will show that in the gauge we used in [3] this term does not contribute to the thermomagnetic coefficient. In a general case, this terms gives a gauge invariant expression for the correlator of electric and heat currents.

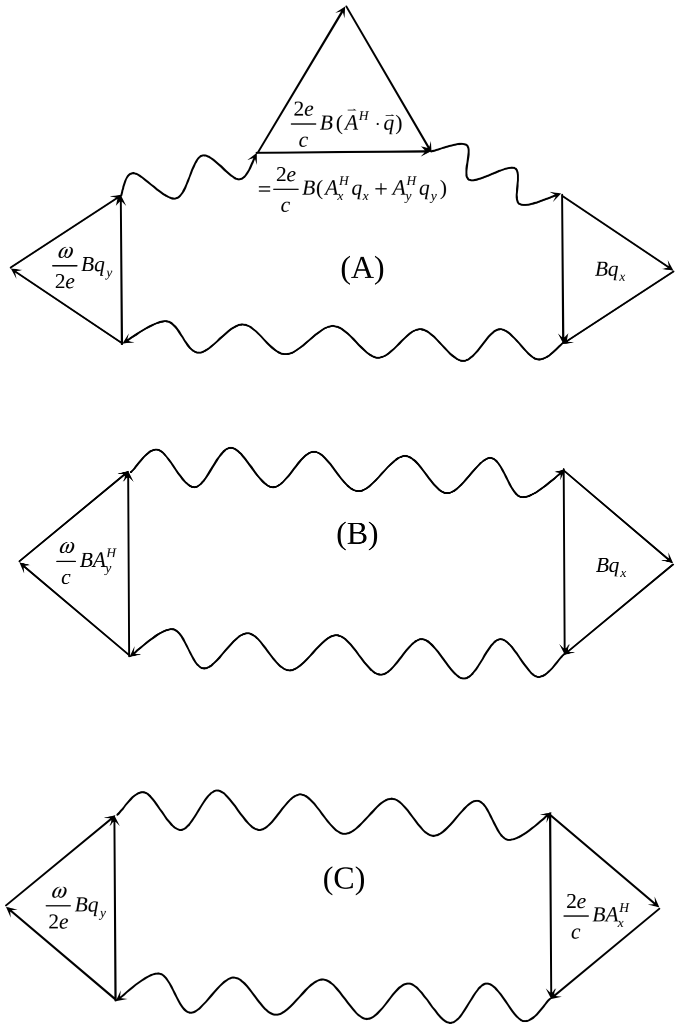
Let us present electric and magnetic fields as

$$\mathbf{E} = i\frac{\Omega}{c}\mathbf{A}^E \qquad \mathbf{H} = i[\mathbf{k}\times\mathbf{A}^H], \tag{3}$$

then the thermal current in the thermomagnetic effect is proportional to

$$\mathbf{E}\times\mathbf{H} = \Omega\mathbf{A}^H(\mathbf{k}\cdot\mathbf{A}^E) - \Omega\mathbf{k}(\mathbf{A}^E\cdot\mathbf{A}^H). \tag{4}$$

FIG. 1: Fluctuation AL diagrams describing the heat current-electric current correlator in crossed electric and magnetic fields. Wavy lines stand for the fluctuation propagators and straight lines stand for the electron Green functions, which form the AL blocks.



As usually, let us put $\mathbf{E}$ along x-axis and $\mathbf{H}$ along z-axis, then $\mathbf{E}\times\mathbf{H}$ will be in the negative direction of y-axis. Then Eq. 4 can be presented as

$$[\mathbf{E}\times\mathbf{H}]_y = -\Omega\mathbf{A}_y^H(\mathbf{k}_x\cdot\mathbf{A}_x^E) + \Omega\mathbf{k}_y(\mathbf{A}_x^E\cdot\mathbf{A}_x^H). \quad (5)$$

To find the thermomagnetic coefficient using the Kubo method, one should calculate the correlator of the heat current directed along y and the electric current directed along x,

$$\mathbf{B}_y^h = \frac{\omega}{2e}B\left(\mathbf{q}+\frac{2e}{c}\mathbf{A}^H\right)_y = \frac{\omega}{2e}Bq_y + \frac{\omega}{c}BA_y^H \quad (6)$$

$$\mathbf{B}_x^e = B\left(\mathbf{q}+\frac{2e}{c}\mathbf{A}^H\right)_x = Bq_x + \frac{2e}{c}BA_x^H. \quad (7)$$

We agree with [1], that calculating thermomagnetic coefficient of fluctuating pairs one should extract A^H from the propagators of Cooper pairs, heat current vertex, and electric current vertex. The corresponding diagrams are presented in Fig. 1. In the diagram (A), A^H is extracted from the propagators, in the diagram (B) it is extracted from the heat current vertex (Eg. 6), and in the diagram (C) it is extracted from the electric current vertex (Eq. 7).

Our opponents accused us in overlooking of the diagram (C) [1]:

"The relevant diagrams for thermoelectric response contain three sources of the vector potential/magnetic-field dependence: (A) Green functions/propagators, (B) heat current vertex, and (C) electric current vertex. The resulting expression is gauge invariant only if all these three sources are taken into account in a consistent fashion and within a specific gauge. In Ref. [3] it is claimed that the contribution (B) cancels the contribution (A) calculated by Ussishkin in Ref. [8]. However the consistency between calculations and gauge choices in the two parts of the same physical quantity is not discussed. Most importantly, contribution (C) is not even mentioned by Sergeev et al., which makes their conclusion erroneous."

We have a simple answer to this criticism. In [3] we used the gauge

$$\mathbf{A}^H = (0, -A^H, 0), \qquad \mathbf{k} = (-k, 0, 0). \quad (8)$$

Obviously, in this gauge the diagram (C) gives zero contribution, because $\mathbf{A}_x^H = 0$. This gauge is widely used for calculations of the Nernst and Hall coefficients, as in this case the second term in Eq. 5 is zero and $[\mathbf{E}\times\mathbf{H}]_y$ is equal to $-\Omega\mathbf{A}_y^H(\mathbf{k}_x\cdot\mathbf{A}_x^E)$ (see Eq. 5).

We can choose another gauge with

$$\mathbf{A}^H = (A^H, 0, 0), \qquad \mathbf{k} = (0, -k, 0). \quad (9)$$

In this case the diagram (B) gives zero contribution, because $\mathbf{A}_y^H = 0$. However, it is easy to see that now the diagram (C) gives exactly the same contributions as the diagram (B) in the previous gauge. Now the term $[\mathbf{E}\times\mathbf{H}]_y$ is given by $\Omega\mathbf{k}_y(\mathbf{A}_x^E\cdot\mathbf{A}_x^H)$ (see Eq. 5).

Obviously, in a general case the diagrams (B) and (C) provide thermoelectric effect, which is proportional to the gauge invariant expression for $[\mathbf{E}\times\mathbf{H}]$ given by Eq. 5.

In conclusion, we explain with all details that in [3] we correctly ignore the diagram (C), because in the gauge we used in [3] the diagram (C) is zero. In any other gauge this diagram gives nonzero contribution. The sum of the contributions of the diagrams (B) and (C) gives the gauge-invariant term, which cancels the contribution of the diagram (A) in zero order in the particle-hole asymmetry (PHA). The nonzero thermomagnetic coefficient arises only in the second order in PHA [3].

Thus, we confirm that in the Fermi liquid with the particle-hole excitations, the thermomagnetic coefficients are always proportional to the square of the particle-hole asymmetry. Therefore, huge thermomagnetic effects observed in high-T_c cuprates can be associated with the larger particle-hole asymmetry due to the Fermi surface

reconstruction or due to a non-Fermi liquid state, such as the vortex liquid.

We are grateful M.N. Serbyn, M.A. Skvortsov, and A.A. Varlamov for detailed discussion of our work [3].

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